

# *Ontario Mathematics Competition*

*Tuesday, October 14, 2025 -  
Friday, October 24, 2025*

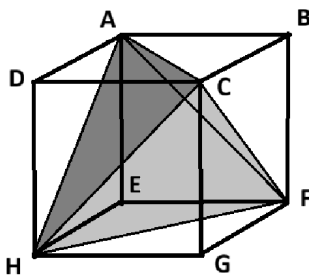
## **General Instructions:**

1. **DO NOT** open the contest booklet until instructed by your proctor.
2. Before the contest begins, the proctors will give you a few minutes to read the instructions and fill in the contestant information section in your bubble sheet. There is no need to rush. Make sure to fill in the required fields legibly.
3. Diagrams are **NOT** drawn to scale. They are intended as aids only.
4. You are allowed scratch paper, a ruler, a compass, and a protractor for rough work.
5. Calculators are permitted as long as they do not have any of the following features:
  - (i) internet access
  - (ii) the ability to communicate with other devices
  - (iii) information previously stored by students (such as formulas, programs, notes, etc.)
  - (iv) a computer algebra system
  - (v) dynamic geometry softwareGraphing calculators (GDCs) are NOT allowed.
6. Please do not discuss the contents of this exam online before 11:59 PM on October 24, 2025.

## **Exam Format:**

1. The OMC consists of twenty-five multiple-choice questions to be completed in 60 minutes.
2. Each question is followed by answers marked A, B, C, D, and E. There is only one correct answer for each question.
3. Scoring: Each correct answer is worth 3 marks.  
There is no penalty for an incorrect answer.  
Each unanswered question is worth 1 mark.  
For tiebreaks, a tiebreaker score will be calculated where a correct answer is worth the same number of marks as its question number.  
For example, question 1 is worth 1 mark.

1. Compute the value of  $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$ .  
A. 45      B. 55      C. 61      D. 62      E. 65
2. What is the second smallest four-digit number divisible by 367?  
A. 1057      B. 1101      C. 1196      D. 1278      E. 1468
3. The operator  $\lambda$  is defined so that for any integers  $a$  and  $b$ , we have  $a\lambda b = a^2 + b$ .  
If  $(x\lambda 8)\lambda 89 = 2025$ , what is the value of  $x$ ?  
A. 6      B. 7      C. 10      D. 44      E. 1928
4. There are 90 students in a class. 6 of them like neither apples or bananas, and 15 of them like both apples and bananas. If there is at least one person who only likes bananas, what is the difference between the largest and smallest possible amount of people who do not like bananas?  
A. 48      B. 56      C. 68      D. 76      E. 78
5. If the lines  $y = 3x + b$  and  $y = 3 - 5x$  intersect at a point  $(p, q)$  such that  $p + q = 11$ , find the value of  $b$ .  
A.  $-3$       B. 1      C. 7      D. 19      E. 25
6. Each of the 6 faces of a cube is randomly labeled with a distinct integer from 1 to 6, inclusive. What is the probability that any two opposite faces are labeled with integers that sum to 7?  
A.  $\frac{1}{6}$       B.  $\frac{1}{12}$       C.  $\frac{1}{15}$       D.  $\frac{1}{24}$       E.  $\frac{1}{30}$
7. The cube  $ABCDEFGH$  has side length 1, as shown in the diagram below. Find the volume of the solid  $ACFEH$ .



- A.  $\frac{1}{4}$       B.  $\frac{1}{3}$       C.  $\frac{2}{5}$       D.  $\frac{4}{9}$       E.  $\frac{1}{2}$
8. Let  $a_1, a_2, a_3, a_4, a_5, a_6, a_7$  be seven pairwise distinct positive integers that form a geometric sequence. Given that the number of positive divisors of each of the numbers  $a_1$  and  $a_7$  is 7. What is the total number of positive divisors of  $a_4$ ?  
A. 7      B. 8      C. 12      D. 16      E. 49

9. The Fibonacci numbers are defined as  $F_1 = 1$ ,  $F_2 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for all integers  $n > 2$ . Let  $S$  be the value of the infinite series

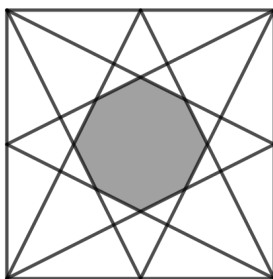
$$\frac{1}{F_1} + \frac{1}{F_2} + \cdots$$

where  $F_n$  is the  $n$ -th Fibonacci number. In which interval does  $S$  lie in?

- A.  $[2, 3)$       B.  $[3, 4)$       C.  $[4, 5)$       D.  $[5, 6)$       E.  $[6, 7)$
10. How many permutations  $(a, b, c, d, e, f, g, h)$  of  $(1, 2, 3, 4, 5, 6, 7, 8)$  satisfy

$$1 < a < b < c < d < e < f < g < h < 8?$$

- A. 45      B. 90      C. 120      D. 180      E. 240
11. In the given figure, the area of the square is 60. By connecting each vertex with the midpoint of the opposite edge, we obtain an octagon. What is the area of this region?



- A. 10      B.  $\frac{50}{9}$       C.  $6\sqrt{2}$       D.  $5 + 3\sqrt{2}$       E. 12
12. Suppose that  $f(x)$  and  $g(x)$  are two quadratic polynomials that satisfy the following conditions:
- The first intersection point between  $f(x)$  and  $g(x)$  is the vertex of  $f(x)$  and lies on the y-axis.
  - The second intersection point between  $f(x)$  and  $g(x)$  is the vertex of  $g(x)$  and lies on the x-axis.
  - The graph of  $f(x)$  can be obtained by rotating  $g(x)$  about the point  $(2, 3)$ .
- What is the value of  $f(1) + g(1)$ ?
- A.  $\frac{27}{8}$       B.  $\frac{9}{2}$       C. 6      D. 9      E.  $\frac{23}{4}$
13. Bowl A contains 147 grams of salt dissolved in 4L of water, whereas Bowl B only contains 3L of pure water. Charles first pours 1L of the water in bowl A to B, then pours 1L of the water in bowl B back to A. After repeating this action an infinite amount of times, how much salt is contained in bowl A?
- A. 73.5      B. 84      C. 94.5      D. 105      E. 126
14. Andy, Benjamin, Carl, David, Edward, and Frank sit in a circle. Andy and Benjamin must sit together, and Carl and Edward cannot sit together. Seating arrangements are considered distinct if one cannot be rotated to match the other. How many distinct seating arrangements are possible?
- A. 6      B. 20      C. 24      D. 72      E. 120
15. If  $a, b$ , and  $c$  are real numbers such that one of them is double of another, find the smallest possible value of  $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ .
- A. 2      B.  $2 + \sqrt{2}$       C.  $2\sqrt{3}$       D. 3      E.  $\frac{1}{2} + 2\sqrt{2}$

16. Link repeatedly flips a fair coin until he gets tails, at which point he stops. If the  $n$ th flip lands on heads, then he earns  $n$  additional rupees. Determine Link's expected total rupees earned from the game.

A. 1      B.  $\frac{3}{2}$       C.  $\frac{5}{3}$       D. 2      E.  $\frac{5}{2}$

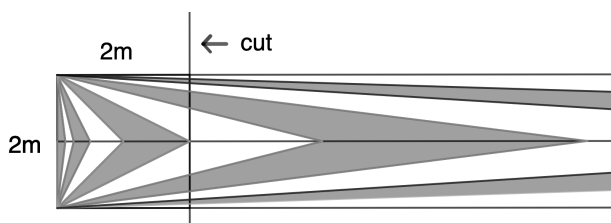
17. A positive integer  $n$  is *powerful* if for any integers  $a, b$ , and  $c$ ,

$$n \mid (a+b)(b+c)(c+a)(a-b)(b-c)(c-a).$$

Compute the sum of all powerful integers.

A. 7      B. 12      C. 28      D. 60      E. 72

18. The Aeldari had a legendary black and white flag of 2 meters wide that extended infinitely to the right, where each region is twice the length of the previous. A reckless officer of theirs, however, accidentally cut the flag in a single slash of his sword into a square with side length 2 meters. Shame! The original flag is shown below, with the pattern continuing infinitely both near the flagpole and into the vastness of space. What is the area of the shaded region left on the square flag?



A.  $\frac{5}{3}$       B. 2      C.  $\frac{23}{12}$       D.  $\frac{9}{4}$       E.  $\frac{5}{2}$

19. There are  $2 \leq n \leq 1000$  bowling pins set up in a straight line. Alex and Ben take turns removing  $k$  initially consecutive pins, where  $k$  is a proper divisor of  $n$ . The person who takes the last pin wins. If Alex goes first, find the sum of all integers  $n$  for which Ben wins.

A. 0      B. 2      C. 38      D. 249999      E. 500499

20. Let  $p(x)$  represent the number  $x$  minus the sum of its digits. Let  $p^k(x)$  be  $p(p(\dots p(x)\dots))$ , where  $p$  is applied to  $x$  a total of  $k$  times. Given that  $5^{10} = 9765625$ , what is the value of  $p^{2025}(5^{10})$ ?

A. 9635058      B. 9667325      C. 9691254      D. 9711836      E. 9751851

21. Let  $ABC$  be a triangle such that  $AB = 7, BC = 24, CA = 25$ . Let  $D, E$  be points on the angle bisector of  $\angle BAC$  such that  $BD$  is parallel to  $EC$ . If the midpoint of  $ED$  is the incenter of  $\triangle ABC$ , what is  $AD$ ? (the incenter is defined as the intersection of the angle bisectors of  $\triangle ABC$ )

A.  $\frac{35}{3}$       B. 12      C.  $\frac{16\sqrt{3}}{3}$       D.  $\frac{15}{2}$       E.  $\frac{60}{7}$

22. The polynomial  $f(x) = x^4 - x^3 + x^2 - x + 100$  has roots  $r_1, r_2, r_3, r_4$ . Let the sequence  $s_n$  denote  $r_1^n + r_2^n + r_3^n + r_4^n$ . Find the last 3 digits of  $s_9 - s_8 + s_7 - s_6 + s_5 - s_4 + s_3 - s_2 + s_1 - s_0$ .

A. 000      B. 167      C. 343      D. 625      E. 797

23. Emma just opened a flower shop and is tracking her hourly sales. She notices an interesting pattern: from one hour to the next starting from hour 1, her sales never drop by more than 1 flower. Suppose that in the 6th hour she sells exactly 4 flowers, how many different possible sales sequences could Emma have had during the first 6 hours?

A. 210      B. 1638      C. 1820      D. 3640      E. 4368

24. Let  $S$  be the sum of all distinct elements in the set

$$\left\{ \left\lfloor \frac{1^2}{101} \right\rfloor, \left\lfloor \frac{2^2}{101} \right\rfloor, \left\lfloor \frac{3^2}{101} \right\rfloor, \dots, \left\lfloor \frac{101^2}{101} \right\rfloor \right\}.$$

What is the remainder when  $S$  is divided by 101?

A. 1      B. 50      C. 51      D. 69      E. 70

25. Blackbeard the pirate has buried his treasure somewhere in the coordinate plane! There's a  $\frac{1}{2}$  chance that the treasure is at  $(0, 0)$ , a  $\frac{1}{4}$  chance that it's at  $(0, 1)$ , and in general a  $2^{-n-1}$  chance that it's at  $(0, n)$  for any positive integer  $n$ . Luffy starts at the point  $(-10, 0)$  and can only move either up or right by 1 unit on each step. What is the expected number of paths that Luffy can take to reach the treasure?

A. 167      B. 343      C. 864      D. 975      E. 1024

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You may use this section for rough work. Only answers filled in the bubblesheet will be evaluated.

Partial marks are not awarded for work shown.



10 A ☐ B ☐ C ☐ D ☐ E ☐    20 A ☐ B ☐ C ☐ D ☐ E ☐