OMC

Ontario Invitational Mathematics Examination

Saturday, November 29, 2025

Contestant Information:

First name

Contestant ID

O						

Last name

General Instructions:

1. **DO NOT** open the contest booklet until instructed by your proctor.
2. Diagrams are **NOT** drawn to scale. They are intended as aids only.
3. You are allowed scratch paper, a ruler, a compass, and a protractor for rough work.
4. Calculators are not permitted during this contest.
5. Express answers as simplified exact numbers as a sum of products unless otherwise indicated. For example, $\pi + 1$ and $1 + \sqrt{2}$ are simplified exact numbers.
6. If the space we provide is not sufficient for you to present your solution, you may use additional blank sheets and label them with your name, contestant ID, and question number.

Exam Format:

1. The OIME consists of two parts Part A and B to be completed in 150 minutes.
   Part A
   1. This part consists of seven questions, each worth 10 marks.
   2. A correct answer is worth full marks, but partial marks may be given only if **relevant** work is shown in the space provided.

   Part B
   1. This part consists of three questions, each worth 10 marks.
   2. Full marks are awarded for a correct answer and clear, complete solutions written in the appropriate location in the answer booklet.

Problem 1

Find all solutions to the equation $(x+5)(x+7)(x+9)(x+11) = -12$. (**Short Answer**)

Your final answer:

Problem 2

The infinite blackboard has all the positive integers written on it, in order. Cauchy erases all multiples of 3 on the blackboard, then Gauss erases all multiples of 5 that are left. Finally, Euler writes all multiples of 7 that have been erased back on the blackboard, in their original positions. What is the 634th number left on the blackboard after the procedure? (**Short Answer**)

Your final answer:

Problem 3

Mark the mosquito is escaping from an angry human! He needs to get from the cube positioned at the back bottom left corner of a $4 \times 5 \times 2$ box to the cube at the farthest corner from his starting position. Each $1 \times 1 \times 1$ cube of the box is connected to each adjacent cube, and Mark can only make moves to the right, forward, up, and down. If he cannot revisit a cube out of fear of being captured, in how many ways can he travel through the 2-cube tall box and get to his destination? (**Short Answer**)

Your final answer:

Problem 4

Find the number of positive integer pairs (m, n) satisfying both of the conditions:

1. $m^2 + n^2 \leq 4050$
2. $m^2 n$ divides $n^3 - m^3$.

(**Short Answer**)

Your final answer:

Problem 5

Let a_n be a sequence such that $a_1 = 1$ and for all positive integers n,

$$a_{n+1} = (-1)^0 \frac{a_n}{n^2} + (-1)^1 \frac{a_{n-1}}{(n-1)^2} + \cdots + (-1)^{n-1} \frac{a_1}{1}.$$

What is a_{2025}? (**Short Answer**)

Your final answer:

Problem 6

Square $ABCD$ of side length 2 has incircle ω. Point E is on line segment AB and F is on line segment BC, such that the line EF is tangent to ω. Find all possible values of the area of $\triangle DEF$. (**Short Answer**)

Your final answer:

Problem 7

On a board consisting of 4 squares aligned horizontally and numbered with $0, 1, 2, 3$ in that order, square x has $x^2 + x$ tokens. Two players make moves in alternating turns such that on each move, a player moves one or more tokens from one square to the square to its left. Once all tokens are in square 0, the game ends and the player who made the last move wins. Harry will move first in one such game. If he and his opponent both play optimally, the move he should begin with is moving x tokens from square y. Find $x \times y$. (**Short Answer**)

Your final answer:

Problem 8

A subsequence is formed by deleting zero or more elements from a sequence without changing the order of the rest. For example, OME is a subsequence of $OIME$, but OEI is not. Over all sequences of length 2025 consisting of only the letters U and W, what is the maximum possible number of subsequences that read UWU? (**Full Solution**)

Problem 9

Triangle $\triangle ABC$ has $\angle ABC = 60^\circ$. Let O and I be the circumcenter and incenter of $\triangle ABC$, respectively. Show that if O and I are distinct, OI is not perpendicular to BI. (**Full Solution**)

[*If you define additional points not presented above, please include a diagram.*]

Problem 10

Determine all positive integers n for which there exists a permutation $(a_1, a_2, \ldots, a_n)$ of $1, 2, \ldots, n$ such that $a_1, 2a_2, \ldots, na_n$ all leave distinct remainders upon division by n. (**Full Solution**)

(Question 10 cont'd)

USE THIS PAGE IF ADDITIONAL SPACE IS REQUIRED

Clearly state the question number being answered and refer the marker to this page.